Please let me (Saul) know if any of the problems are unclear or have typos. There are no problems to turn in from this exercise sheet.

Exercise 4.1. For each of the following groups, verify its growth rate.

1. $H_3(\mathbb{Z})$ has $\gamma(n) \sim n^4$.
2. $BS(1, 2)$ has $\gamma(n) \sim \exp(n)$.
3. $F(a, b)$ has $\gamma(n) \sim \exp(n)$.

Exercise 4.2. Suppose that G is a finitely generated group. Then G has sublinear growth if and only if G is finite.

Exercise 4.3. Determine if $BS(1, 2)$ is nilpotent.

Exercise 4.4. Define $H_d(\mathbb{Z})$ to be the set of upper triangular matrices, over the integers, having only ones on their diagonal.

1. Show that $H_d(\mathbb{Z})$ is a group under matrix multiplication.
2. Prove that $H_d(\mathbb{Z})$ is torsion free.
3. Prove that $H_d(\mathbb{Z})$ is nilpotent of class $d - 1$.

Exercise 4.5. Suppose that $H < G$ is a finite index subgroup.

1. Prove that G is finitely generated if and only if H is.
2. Prove that $\gamma_G \sim \gamma_H$: that is, H and G have comparable growth rates.

Exercise 4.6. Suppose that Γ is a connected, infinite, locally finite graph. Prove that Γ contains a geodesic ray. (And in fact, one based at any desired vertex.)

Exercise 4.7. Suppose that G is a group, finitely generated by S. Suppose that $\Gamma_S = \Gamma(G, S)$ is its Cayley graph. Prove that Γ_S contains a geodesic line.