Please let me (Saul) know if any of the problems are unclear or have typos.

Exercise 5.1. Suppose that $U \subset \mathbb{C}$ is a domain. We call a contour in U *semicircular* if it is simple, closed, and its pieces are either line segments or arcs of circles. Suppose that $R \subset U$ is compact, is nonempty, equals the closure of its interior, and has boundary which is a disjoint union of semicircular contours. Then we call R a *semicircular region*. Give a direct proof that R admits a triangulation. ◇

Exercise 5.2. Suppose that $\gamma\colon [-\pi/2, \pi/2] \to \mathbb{C}$ is defined by $\gamma(\theta) = \frac{3}{2}(i + e^{i\theta})$.

a) Sketch γ and indicate its orientation.

b) Compute the contour integral $\int_\gamma \frac{dz}{z-i}$.

c) Carefully justify your answer, giving labelled sketches as needed. ◇

Exercise 5.3. Recall that $\textsc{Exp}\colon \mathbb{C} \to \mathbb{C}$ is the complex exponential. For any $z \in \mathbb{C}$ we define:

$$f(z) = \int_0^1 \frac{\textsc{Exp}(tz) - 1}{t}\, dt$$

Prove that $f\colon \mathbb{C} \to \mathbb{C}$ is well-defined. Compute $f(0)$ and $f(1)$ (the latter to five decimal places of accuracy). Prove that f is holomorphic. ◇

Exercise 5.4. Suppose that $U \subset \mathbb{C}$ is a domain. Suppose that z_0 is a point of U . Suppose that $f, g\colon U \to \mathbb{C}$ are holomorphic. Recall that $\textsc{Ord}(f, z_0)$ is the *order of vanishing* of f at z_0. Prove the following.

a) $\textsc{Ord}((z - z_0)^N, z_0) = N$ for $N \geq 0$ an integer.

b) $\textsc{Ord}(f \cdot g, z_0) = \textsc{Ord}(f, z_0) + \textsc{Ord}(g, z_0)$.

c) $\textsc{Ord}(f + g, z_0) \geq \min\{\textsc{Ord}(f, z_0), \textsc{Ord}(g, z_0)\}$. ◇

Exercise 5.5. Suppose that $U \subset \mathbb{C}$ is a domain. Suppose that $Z \subset U$ is isolated in U. Suppose that $K \subset U$ is compact. Prove that $Z \cap K$ is finite. ◇