If you have questions about this exam feedback, please ask on the Q&A forum. Here is a "stem and leaf plot" of the (unscaled, unchecked) marks.

20		9
30		9 9 9 9
40	0 1 2 2 3 3 4 4 4 4 4 4	5 5 6 6 6 7 7 8
50	1 1 1 2 2 3 3 3 4	5 5 5 6 6 6 6 7 7 7 7 7 8 8 9
60	0 0 0 1 2 2 2	5 5 6 7 8 9
70	0 0 1 1 2 2 3 4 4 4	5 5 5 5 5 6 6 7 8 8
80	0 0 1 1 2 2 2 3 3 3 4 4	7 8 8 9 9
90	3 3 3 3	6 6

The min/max mark was 29/96. Of the 106 scripts, 25%/50%/75% received marks at or below 51/61.5/76. The mean was 63.8 and the standard deviation was 16.4.

Question 1:

a) Several students gave the following incorrect chain of reasoning:

$$z = x + iy, \qquad \text{so} \qquad dz/dx = 1, \qquad \text{so} \qquad dz = dx.$$

At least one student treated z, x, y, r, and θ as functions of new variable t. This works and is a nice way to think about it.

c) We need to indicate the region D with a bit of shading and/or a label.

e) It is a bit circular to answer this question using the "complex analysis version of the fundamental theorem of calculus".

Only a handful of students used, here, the fact that $dz/z = dr/r + i\,d\theta$.

f) Several students correctly wrote that the integral is four times $\log(i)$. They then incorrectly deduced that $4 \cdot \log(i) = \log(i^4) = \log(1) = 0$.

Several students incorrectly used the fundamental theorem of calculus to deduce that the integral is $\log(1) - \log(1) = 0$.

Question 2:

c) Several students incorrectly used the Cauchy–Riemann equations.

d) At least one student integrated around the unit circle and applied Cauchy's theorem! In fairness, the algebra (this way) is easier...

g) Some students incorrectly claimed that the product of a holomorphic function and a non-holomorphic function is again non-holomorphic.

At least one student answered this using the open mapping theorem! Another used the identity theorem.

Question 3:

a) It is a mistake to not mention the choice of region. Also, the poles need to miss the contour. Some students, instead of using regions, incorrectly talked about the "interior" of a contour.

b) Many students noted that the function $x/(1+x^2)$ is odd. They correctly deduced that its integral from $-R$ to R is zero. They then incorrectly claimed that the integral over the real line converges.

Several students incorrectly said that $\int_1^\infty dx/x$ converges.

Several students incorrectly said that $\int_0^\infty dx/x^{3/2}$ converges.

c) A meromorphic function requires, as part of its definition, a choice of domain. Also, $\sqrt{z}$ is not meromorphic at the origin. Finally, the choice of branch needs to agree with the original (real) integrand on the positive reals.

d) It is a mistake for the contour to run through the origin, to run through the point i, or to include a very small circle about the point i.

e) Strictly speaking, we need to check that a pole w of a meromorphic function f is a simple pole before using the following:

$$\textsc{Res}(f, w) = \lim_{z \to w} (z - w) \cdot f(z)$$

No points were deducted for failing to check simplicity here.

h) Without computing anything, we know that J is real and positive. This observation catches many mistakes.

At least two students used the substitution $x = u^2$ (so $dx = 2u\,du$) to prove that $J = 2\int_0^\infty \frac{u^2}{1+u^4}\,du$. This gets rid of the branch point at the origin, so we no longer need a "keyhole" contour. This is a nice simplification of the problem.

Question 4:

a ii) Three marks deserve three sentences.

b) Partial marks were given to answers beginning with an incorrect drawing of W.

Several students incorrectly claimed that the map $z \mapsto z^8$ gives a biholomorphism between W and $\mathbb{D}$.

Several students incorrectly claimed that the map $r\exp(i\theta) \mapsto r\exp(2i\theta)$ is holomorphic.

Several students incorrectly claimed that the map $r\exp(i\theta) \mapsto \frac{r}{1-r}\exp(i\theta)$ is holomorphic.

Many students incorrectly "found" a linear fractional transformation transforming the upper half disk into the upper half plane.