2025-10-31 LECTURE 12 MA3B8 SAUL SCHLEIMER

(1) TWO KINDS of BOUNDARY

TOPOLOGICAL: SUPPOSE X IS A TOP SPACE. SUPPOSE $A \subset X$

DEFINE ∂A = CLOSURE(A) - INTERIOR(A)

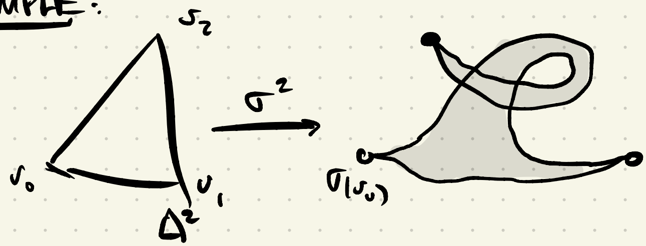
$= \bar{A} - A^\circ$

EXAMPLE: $\partial \mathbb{D} = S^1$.

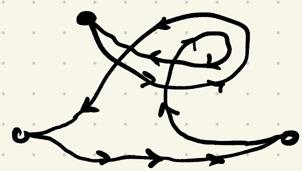
HOMOLOGICAL: SUPPOSE $c \in C_n(X)$ IS AN n-CHAIN.

SAY $c = \sum_i a_k \sigma^n_k$. THEN $\partial c = \sum_i a_k \partial \sigma^n_k$.

EXAMPLE:



SO; $\partial\sigma$ IS A 1-CHAIN



SO THE HOMOLOGICAL BOUNDARY NEED NOT PARAMETRISE THE TOPOLOGICAL BOUNDARY.

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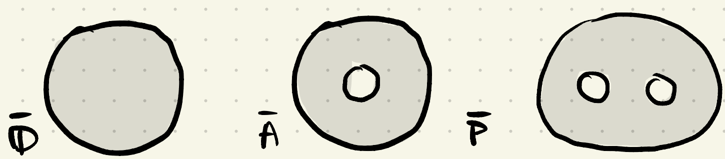
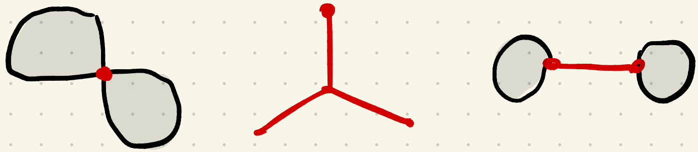
(2) REGIONS AND TRIANGULATIONS.

DEF: SUPPOSE $R \subset \mathbb{C}$ IS COMPACT, ∂R IS A COLLECTION of EMBEDDED CONTOURS, AND EVERY $p \in \partial R$ LIES IN THE CLOSURE of THE INTEROR of R.
THEN WE CALL R A REGION IN $\mathbb{C}$

EXAMPLE:

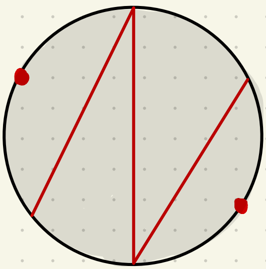


NONEXAMPLES:



DEF: (INFORMAL) SUPPOSE $R \subset \mathbb{C}$ IS A REGION. SUPPOSE $c = \sum_1^{\ell} a_k \sigma_k^2 \in C_2(\mathbb{C})$. SAY THE CHAIN c **TRIANGULATES** R IF THE UNION of THE IMAGES of THE σ_k^2 NICELY PARTITION R. [(*) AND INDUCE STANDARD ORIENTATION ON ∂R AND ARE C^1 ON ∂R.]

PICTURE



$R = \overline{\mathbb{D}}$ TRIANGULATED BY FOUR TRIANGLES.

LEMMA: SUPPOSE c TRIANGULATES R. THEN ∂c PARAMETRISES ∂R. $\square$

COROLLARY (of CAUCHY'S THM). SUPPOSE $R \subset U$ A REGION. SUPPOSE $f: U \longrightarrow \mathbb{C}$ HOLOMORPHIC. THEN $\int_{\partial R} f \, dz = 0$. ["HOLOMORPHIC FCNS INTEGRATE TO ZERO ABOUT REGIONS"]