

Please let me (Saul) know if any of the problems are unclear or have typos. There are no exercises due this week.

**Exercise 2.1.** Suppose that  $F: X \times I \rightarrow Y$  is continuous. For each  $t \in I$  define  $f_t: X \rightarrow Y$  by  $f_t(x) = F(x, t)$ . Prove that  $f_t$  is continuous.

**Exercise 2.2.** Show that the relation  $f \simeq g$  of being homotopic is an equivalence relation on maps.

**Exercise 2.3.** Show that  $f \simeq g$  implies  $h \circ f \simeq h \circ g$  (assuming that all compositions make sense). Show that the relation  $X \simeq Y$  of being homotopy equivalent is an equivalence relation on topological spaces.

**Exercise 2.4.** Show that  $\mathbb{R}^n - \{0\} \cong S^{n-1} \times \mathbb{R} \simeq S^{n-1}$ . That is, the first pair of spaces are homeomorphic while the second pair are homotopy equivalent. Use this to prove that  $\mathbb{R}^n - \{0\} \simeq S^{n-1}$ .

**Exercise 2.5.** Fix  $m, n \in \mathbb{N}$  so that  $0 < n < m$ . We embed  $\mathbb{R}^n$  into  $\mathbb{R}^m$  by taking  $(x_1, \dots, x_n) \in \mathbb{R}^n$  to  $(x_1, \dots, x_n, 0, \dots, 0) \in \mathbb{R}^m$ . Show that  $\mathbb{R}^m - \mathbb{R}^n \simeq S^{m-n-1}$ .

**Exercise 2.6.** [Medium.] Consider the capital letters of the alphabet A, B, C, ... in a sans serif font. Each of these gives a graph in the plane. Sort these into homotopy equivalence classes. Clearly state any unproven assumptions that you rely on.

**Exercise 2.7.** [Medium.] Show that the eyeglasses graph  $E$  and the theta graph  $T$  (both shown in Figure 2.8) are homotopy equivalent.

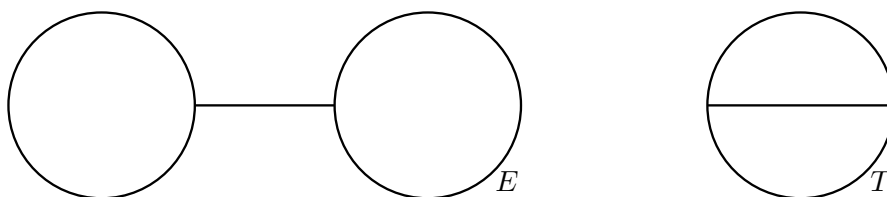


Figure 2.8: Left: the eyeglasses graph. Right: the theta graph.

**Exercise 2.9.** Define  $\omega_n: I \rightarrow S^1$  by  $\omega_n(t) = \exp(2\pi i n t)$ . Show that the concatenation  $\omega_p * \omega_q$  is homotopic, rel endpoints, to  $\omega_{p+q}$ . (First work out the special case of  $p = 2$  and  $q = 1$ . Then do the general case.)