MA3F2 Exercise sheet 6.

These exercises are mainly taken from the sixth week's lectures. Please let me know if any of the problems are unclear or have typos.

Exercise 6.1. Which knots up to seven crossings are two-bridge? (You may find useful the theorem proved in class that a knot is 2 bridge if and only if it is a four-plat.)

Exercise 6.2. Starting with the standard diagrams find Schubert normal form for the trefoil, figure eight, Hopf link, and Whitehead link. Find Schubert normal form for the (2, p)-torus links. There will be a nice pattern to these two-bridge positions. [Harder: What is Schubert normal form for the (3, p)-torus links?]

Exercise 6.3. Show that the (2, p) and the (2, q)-torus links are isotopic if and only if p = q.

Exercise 6.4. [Hard] All 2-bridge knots have an alternating diagram. (Hint: as usual it is easier to work with 4-plat diagrams.)

Exercise 6.5. Check that, with the conventions given in class, the map $\pi: B_n \to S_n$ is an antihomomorphism. That is, $\pi(\sigma \cdot \tau) = \pi(\tau) \circ \pi(\sigma)$.

Exercise 6.6. Show that twice the braid index of a link is an upper bound for the bridge number. Show that this bound is not sharp.

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